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ABSTRACT

This study was conducted to address two questions: (1) are deficiencies in fraction skills due to current instructional programs, and, if yes, (2) should instruction on fractions be postponed until secondary school or should instruction at the elementary levels be revised? A battery of fraction tests consisting of 78 items was developed; tests were classified according to cognitive level as computation, comprehension, or applications. Tests were given to average eighth-grade students. More than 1500 students took 10-item parts of the battery. Results indicated that students do not have the fractional skills needed to compute or to solve problems. Responses to number line and diagramatic items indicate they understand fractional concepts; however, performance on other items indicates a poor understanding of the structure of the rational number system. The investigators recommended that some instruction on fractions be postponed. (SD)

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SMESG WORKING PAPER NO. 20

A Survey of Student Achievement with Fractions

By: Joan Ginther, Katie Ng, and E. G. Begle

October 1976

Introduction

In most schools in the United States, some intuitive ideas about fractions are introduced in the primary grades. In the upper elementary grades it is expected that students will learn how to compute with fractions and to solve word problems involving fractions. Usually, some revision and reinforcement of fraction skills and understandings are provided in junior high school, mainly in grade seven.

In general, teachers find fractions a hard topic to teach and students find fractions a hard topic to learn. Our journals have reported numerous experiments on different ways to teach the four operations on fractions and have also provided many discussions of the proper grade placement of the various aspects of the fraction concept.

Most classroom teachers and most educational researchers in this country seem to believe that the present grade placement of fraction topics need little change, and that we should concentrate on improving our instructional procedures at each grade level. Others, however, believe that fractions are much too complex and difficult a topic for elementary school students and that substantial work with fractions should be postponed until the student has been introduced to the basic concepts of algebra.

Thus there are two questions which need answers. Are deficiencies in fraction skills and understandings due to our instructional programs, or are they due to other factors not under the control of the school? Second, if the answer to this first question is that the instructional programs are ineffective, should we concentrate our energies on trying to improve them or should we plan to postpone much of the rational number curriculum to secondary school?

To date, discussion about these questions has been hampered by our lack of hard information as to how well our present practices are working. A good deal of information about student achievement in fraction skills and understandings was gathered by the National Longitudinal Study of Mathematical Abilities (NLSMA), but that information was gathered over a decade ago, and much could have changed since then.

The National Assessment of Educational Programs provided, in its first look at mathematics achievement, only a modicum of information about fractions.

The purpose of this study, then, was to gather such information. It appeared that what would be most useful would be information about how much average students can learn about fractions under the best conditions. Accordingly, a number of junior high schools were sought in well-to-do suburbs of large cities, in university towns, and, in general, in areas where any weaknesses in the instructional program could not be ascribed to such factors as poverty, ethnicity, etc., but only to the program itself. A very gratifying number of invitations to such schools to participate in the study were accepted. A list of these schools appears in the Appendix, together with the names of the persons who provided the cooperation.

Method

For this study a battery of fractions tests were prepared. The individual tests were classified as being at the cognitive levels of computation, or comprehension, or application and are listed below. Most, but not all, of the items were chosen from the NLSMA test batteries.

The entire battery, which was administered early in 1976, contained 78 items. In order to decrease the inconvenience to students and teachers, the items were apportioned among 13 different test forms, each of which had fewer than 10 items all printed on one page. These forms were distributed randomly within each class participating in the study. The minimum number of students attempting a particular form was 161, the maximum was 203, and the average was 193.

The plan was to administer these tests only to average students. Each participating teacher was asked to return, along with the completed tests, his estimate of the average IQ of the class. Only when the estimated average IQ was between 90 and 115 were the scores included in the anlaysis of the data of the 95 classes included. The average estimated IQ was 105.6.

Only eighth grade students were tested on the grounds that these students would, for the most part, have completed their work on fractions

but would not have had time to forget much of what they had learned. Consequently, the results, to which we next turn, should give us a good estimate of what average students can learn about fractions in our most advantaged schools and therefore an upper bound to what we can expect from the average U. S. school.

Results

A. Computation. There were five computation tests, each containing five items. We give a brief description of each test and reproduce the second easiest and the second hardest item of each test together with the percentage of correct answers.

Addition. Five addition items, all in vertical form, for which the student was to write the answer.

$$\begin{array}{r} \frac{1}{6} \\ +\ \frac{5}{8} \\ \hline \end{array} \quad (63\%) \qquad\qquad \begin{array}{r} \frac{3}{8} \\ +\ \frac{1}{2} \\ \hline \end{array} \quad (56\%)$$

Subtraction. Five items, all in vertical form, for which the student was to write in the answer.

$$\begin{array}{r} \frac{3}{4} \\ -\ \frac{2}{5} \\ \hline \end{array} \quad (58\%) \qquad\qquad \begin{array}{r} 1\,\frac{1}{8} \\ -\ \frac{1}{2} \\ \hline \end{array} \quad (53\%)$$

Multiplication. Five items, all in horizontal form, for which the student was to write in the answer.

$$\frac{5}{8} \times 32 = \underline{\qquad} \quad (65\%) \qquad \left(\frac{7}{3} \times \frac{5}{14}\right) \times \frac{9}{10} = \underline{\qquad} \quad (51\%)$$

Division. Five items, all in horizontal form, for which the student was to write in the answer.

$\frac{7}{8} \div \frac{5}{16} =$ _____ (40%)	$7\frac{1}{3} \div 2\frac{1}{5} =$ _____ (35%)

Lowest Terms. Five items, each consisting of a fraction to be rewritten in lowest terms.

$\frac{6}{8}$ (93%)	$\frac{9}{18}$ (82%)

In scoring the first four of the above tests, answers not in lowest terms were accepted if correct. Scores on the fifth test indicate that students are quite good at reducing fractions when asked to. It is therefore interesting to note that in many of the problems in the first four tests a substantial number of the answers, sometimes more than half, were left unreduced.

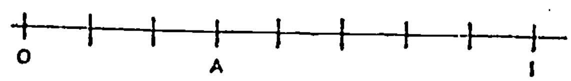
B. Comprehension. A variety of tests were at this cognitive level.

Structure. Five items which could be answered very easily by a student who understood the structure of the rational number system. In each item a number was to be filled in to make a true statement.

$2 \times \square = 1$ (42%)	$\frac{1}{3} \times \square \times 5 = 5$ (30%)

Number Line. Four items asking about the relationship between fractions and points on a number line. These were in multiple choice format.

In these two questions you are asked about a point on a number line and the fraction it represents. Circle the letter in front of your answer choice.

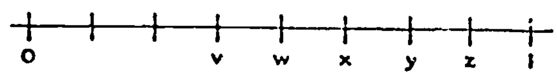
1) A is 

(62%)

(A) $\frac{1}{2}$ (D) $\frac{1}{3}$

(B) $\frac{1}{4}$ (E) $\frac{5}{8}$

(C) $\frac{3}{8}$

2) Which is $\frac{3}{4}$? 

(A) v (D) y

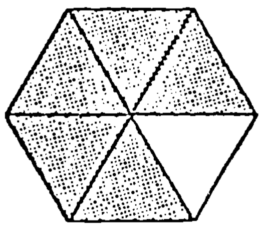
(B) w (E) z

(C) x

(48%)

Fractions and Diagrams. Fifteen items, in multiple choice format, asking about the relationships between geometric figures and fractions. The fifth easiest and fifth hardest items are reproduced.

In each of these three questions there is a drawing on the left. Part of the drawing is shaded. The drawing suggests a fractional number. You are to choose the fraction on the right which names the same fractional number as the shaded part of the drawing. Circle the letter in front of your answer choice.



(A) $\frac{1}{6}$

(B) $\frac{3}{6}$

(C) $\frac{5}{6}$

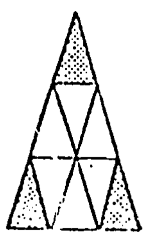
(D) $\frac{7}{6}$

(E) None of these

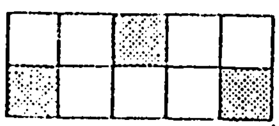
(88%)

$\frac{1}{3}$

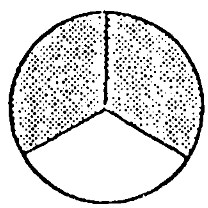
(A)



(B)



(C)



(D)

(E) None of these

(37%)

C. Applications.

Working with Fractions. Twelve problems about fractions requiring more than rote computation. Each was in multiple choice format. This set of items had been shown in other SMESG studies to be a good predictor of mathematics achievement. The fourth easiest and fourth hardest of these are reproduced here.

The inverse of multiplying by 3 is

A) adding B) subtracting C) multiplying by 3
D) dividing by 3 E) none of these (65%)

How many twelfths does $2\frac{1}{4}$ equal?

A). 28 B) 27 C) 25 D) 16 E) 12 (40%)

Word Problems. Twelve word problems, with the answer to be written in. The fourth easiest and the fourth hardest are reproduced here.

A girl weighs $64\frac{1}{2}$ pounds. Her brother weights $\frac{1}{2}$ as much as she weights. How many pounds does he weigh? ..... ANS (54%)

One-half the students of a school are going to a concert. These students will be taken in 5 buses. What fraction of the students of the school will ride in each bus? .... ANS (36%)

D. Justification of Division Algorithms. Most of the usual methods of justifying the algorithm for division of fractions were included in this item, which was included in eight of the thirteen forms. The order of the first four responses was rotated from form to form. Of the 1570 students who looked at this item 102 (6%) made no response.

Here is a division problem. $\boxed{\frac{1}{2} \div \frac{1}{3} = ?}$

Circle the letter in front of the statement below which best explains why $\frac{3}{2}$ is the correct answer to this division problem.

A) To divide fractions, you invert and multiply.

B) $\frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$

C) $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$, so $\frac{1}{2} \div \frac{1}{3} = \frac{3}{6} \div \frac{2}{6} = 3 \div 2 = \frac{3}{2}$

D) $\frac{1}{2} \div \frac{1}{3} = \dfrac{\frac{1}{2}}{\frac{1}{3}} = \dfrac{3 \times \frac{1}{2}}{3 \times \frac{1}{3}} = \dfrac{\frac{3}{2}}{1} = \frac{3}{2}$

E) None of the above. The correct answer is $\frac{2}{3}$.

The 1468 responses to this item were distributed as follows:

A) 50%

B) 7%

C) 11%

D) 15%

E) 17%

Discussion. These results provide a clear answer to our first question. Our present instructional program does not provide our students, even in the best of schools, with the fractional skills, in computation and in problem solving, that they should have. The computation results, for the students included in this study, are clearly unsatisfactory. Their performance on relatively simple word problems was also quite disappointing.

Our second question, however, does not have such a clear cut answer. At first glance, it would not seem unreasonable for us to throw up our hands and declare that most of the work on fractions should be postponed to secondary school -- it is too hard for elementary students.

However, a second look at the results listed above suggests that the situation may not be so clear cut. One of the major objectives of most of the curriculum projects of the sixties was to teach mathematics meaningfully. This meant making sure that the students understood the basic concepts. It also meant making sure that they understood how the basic concepts of a mathematical system were interrelated (i.e. the structure of the system). Finally, it meant developing and justifying algorithms, on the basis of concepts and structure, before the students were asked to practice them.

The results listed above for the Number Line and the Fractions and Diagrams tests do indicate that students today have a reasonable understanding of the fraction concept. However, the low scores on the Structure test indicate a poor understanding of the structure of the rational number system. Also, the responses to the division algorithm item suggest that the majority of students have been taught this algorithm in a rote fashion.

Whether our elementary and junior high school teachers could teach fractions in a more meaningful way than they now do, and whether the result would be improved student learning of fractions, are questions that can only be answered by controlled experimentation. But until these last questions are answered affirmatively, our original answer to the second main question stands. Much of the work on fractions should be postponed to secondary school.

<u>Postscript on Metrics</u>. The U. S. is moving slowly but surely to the metric system of measurement. Many mathematics educators believe that once the change has been made, much less work on fractions will need to be done in our schools. We do not share this opinion.

While it may be possible to reduce the amount of practice in computation, a good understanding of fractions and the ability to translate real problems into mathematical problems involving fractions will continue to be important goals of general education. Many situations in the world around us lead to fractions whose denominators are not powers of ten. A day will continue to be one seventh of a week and the aces will continue to constitute one thirteenth of a deck of cards. And even when a measurement system is decimalized we may find it convenient to use non-decimal fractions. We usually call our twenty five cent coin a quarter.

Our recommendation above is that some aspects of fractions be postponed, not eliminated. It is made in full realization of the move toward metric measurement.

APPENDIX

Neil Cummins School Corte Madera, CA 94925	Mrs. Mary Jo Ward
Sinaloa Junior High Novato, CA 94947	W. L. Metteer
Greenwich Public Schools Greenwich, CT 06830	Dr. Murray Stock
Western Junior High School Greenwich, CT 06830	Dr. Murray Stock
Central Junior High School Greenwich, CT 06830	Dr. Murray Stock
Eastern Junior High School Greenwich, CT 06830	Dr. Murray Stock
Augusta Raa Middle School Tallahassee, FL 32303	Beth Cannon
Developmental Research School Tallahassee, FL 32306	Jerry Whitmore
Burney Harris Middle School Athens, GA 30601	Ellen Hanna
Hilsman Middle School Athens, CA 30601	Peggy Neal
University Middle School Bloomington, IN 47401	Dr. Dale Glenn
Clay Junior High Carmel, IN 46032	S. B. Robinson
Tilden Junior High Rockville, MD 20852	Judith M. Cire
E. Brooke Lee Junior High Silver Spring, MD 20902	Milton Ruark
Belt Junior High Wheaton, MD 20906	Barbara Pond
Bloomfield Hills Junior High Bloomfield Hills, MI 48013	John Cullen

Hannah Middle School East Lansing, MI 48823	Tony Egnatuk
McDonald Middle School East Lansing, MI 48823	Dr. S. V. DiFranco
Holt Junior High Holt, MI 48842	Tom Horan
West Junior High Rochester, MI 48063	Douglas Treais
Birney Junior High Southfield, MI 48075	John Caekaway
Thompson Junior High School Southfield, MI 48075	Karl Buttenmiller
Ridgeview Junior High Columbus, OH 43214	Jerry Martin
Irving Middle School Norman, OK 73069	Jennifer Sprague
Hefner Junior High School Oklahoma City, OK 73132	Norman Dillard
Hollidaysburg Area Junior High Hollidaysburg, PA 16648	William V. Waryck
Knox County Schools Knoxville, TN 37902	Charleen DeRidder
Oak Ridge Schools Oak Ridge, TN 37830	Edward Smith
Burnet Junior High Austin, TX 78758	Mary Schroeder
Dobie Junior High Austin, TX 78753	Mary Schawe
Murchison Junior High Austin, TX 78731	Jamelle Bishop
Porter Junior High Austin, TX 78704	Charles Cauley
Highline Public Schools Seattle, WA 98166	Charles Hardy